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Invited Lecture

Fluctuation absorption of sound in smectics

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A comparison is made of the low-frequency dynamics of smectics A and smectics C. It is shown that in both cases fluctuations of smectic layer displacements bring about contributions to the bulk viscosity coefficients (diverging as ω^{-1}) and consequently lead to anomalous sound attenuation. However, the coefficients of these fluctuation contributions differ greatly between smectics A and smectics C. The reason for this is a strong coupling of the orientational mode in smectics C to layer displacements (the so-called undulation mode). The results of this work make it possible to give a complete interpretation of the experimental data on sound absorption in smectics.

1. Introduction

Smectics are liquid-crystalline phases with one dimensional density modulation. As was shown by Landau and Peierls [1] in the nineteen thirties, long wavelength fluctuations destroy genuine long range order. Nevertheless, elasticity of smectic layers is a properly defined quantity. As has been demonstrated by Grinstein and Pelcovits and one of the authors (E.K.) [2, 3], the respective elasticity moduli are only weakly (logarithmically) scale dependent.

More explicitly, long wavelength fluctuations of smectic layers manifest themselves in the dynamics of smectics. As was first shown by Mazenko, Ramaswamy and Toner [4], fluctuations lead to corrections, diverging as ω^{-1}, to the so-called smectic bulk viscosity coefficients. This gives rise to an anomalous dependence of the attenuation of both first and second sound. A consistent theory of dynamic fluctuation effects in smectics, taking into account logarithmic renormalizations of the parameters, has been constructed by two of the authors (E.K. and V.L.) [5].

Strictly speaking, these works concern the smectic A phase. Therefore the problem remains of the peculiarities of the anomalous attenuation of sound in the smectic B and C phases, where there is an additional orientational mode (compared with the S_A phase). This problem for the S_B phase has been investigated in [6], where the authors came to the conclusion that the expressions describing the anomalous attenuation of sound in the S_B and S_A phases practically coincide. However, the corresponding problem for the S_C phase requires a more detailed analysis.

In the study of the spectrum of smectics B it has been assumed that coupling of the orientational mode and smectic layer displacements (the undulation mode) is weak. This coupling no doubt is missing in the S_B phase. However, the analysis of the critical dynamics of the smectic A–smectic C transition performed by us in [7] shows that, despite the fact that the tilt of the director with respect to the layers of the S_C

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phase is small, the coupling of the orientational and undulation modes in the S_C phase is not at all weak.

Collin *et al.* [8] have observed anomalous attenuation of sound in the S_C phase. It is important that they had previously observed anomalous attenuation of sound in the S_A phase of the same substance [9], which allows comparison of the character of the attenuation in the S_A and S_C phases. This prompted us to perform a consistent calculation of the anomalous attenuation in the S_C phase.

2. The smectic phases

The layered structure of smectics is conveniently described by the density-modulation phase. We shall denote this variable by W. By virtue of the definition of W, the condition $W(t, \mathbf{r}) =$ constant fixes the position in space and evolution in time of a certain smectic layer. Correspondingly, the vector ∇W fixes the direction of the normal to the layer, and

$$\mathbf{l} = \nabla W/|\nabla W| \tag{1}$$

is a unit vector along this direction.

The anisotropy of a liquid crystal is characterized by the director $\mathbf{n}$, pointing to the preferred direction of the long axes of molecules. In smectics A the director is perpendicular to the layer, i.e. it coincides with the vector $\mathbf{l}$. In smectics C the director is tilted by a certain angle to the normal; therefore the vector

$$\boldsymbol{\psi} = \mathbf{n} \times \mathbf{l} \tag{2}$$

is non-zero. The magnitude of the vector $\boldsymbol{\psi}$ determines the tilt angle of the director, and the unit vector

$$\mathbf{n}_1 = \boldsymbol{\psi}/|\boldsymbol{\psi}| \tag{3}$$

fixes a preferred direction in the plane of the smectic layer.

In the S_C phase the magnitude $|\boldsymbol{\psi}|$ is fixed, whereas the direction of the vector $\mathbf{n}_1$ is not. Thus the macroscopic state of smectics C should be characterized by an extra variable (compared with the S_A phase), describing the direction of $\mathbf{n}_1$. This is accounted for by spontaneous breaking of the rotational symmetry of a layer in smectics C.

By virtue of the conditions $|\mathbf{n}_1|^2 = 1$ and $\mathbf{n}_1 \cdot \mathbf{l} = 0$, the vector $\mathbf{n}_1$ has only one degree of freedom, which we shall term orientational. This degree of freedom can be conveniently described by means of the angle φ, whose variation is determined by

$$\delta\varphi = 2(\mathbf{n}_1 \times \mathbf{l}) \cdot \delta\mathbf{n}_1. \tag{4}$$

Apart from the variables W and φ, a complete set of long wavelength variables of smectics C also involves the momentum density $\mathbf{j}$, the mass density ϱ and the specific entropy σ. Thus the energy of the S_C phase can be written as

$$\mathscr{H} = \int d^3r \left[\frac{|\mathbf{j}|^2}{2\varrho} + E(\varrho, \sigma, \nabla W, \nabla_i \nabla_k W, \mathbf{n}_1, \nabla\varrho) \right]. \tag{5}$$

The energy density E depends only on derivatives of W (since W is defined up to a constant) and the gradient of $\mathbf{n}_1$ is expressed in terms of the gradients of φ and W. The pressure P is expressed in terms of the energy density as

$$P = \varrho \frac{\partial E}{\partial \varrho} - E. \tag{6}$$

In the present approximation the energy density E is represented as

$$E = \varepsilon(\varrho, \sigma) + E_W + E_\varphi. \tag{7}$$

The smectic part of the energy is

$$E_W = \tfrac{1}{8}B(q_s^{-2}(\nabla W)^2 - 1)^2 + \tfrac{1}{2}Kq_s^{-2}(\nabla^2 W)^2. \tag{8}$$

Here B is the modulus of compressibilities of the smectic layer, K is the elasticity coefficient, which is of the order of magnitude of the Frank modulus, and q_s is the wavevector, determining the density modulation period (this period is equal to $2\pi q_s^{-1}$). The orientational energy is written as

$$E_\varphi = \tfrac{1}{2}[\alpha_1\delta_{ik}^\perp + (\alpha_2 - \alpha_1)n_{1i}n_{1k} + \alpha_3 l_i l_k]\nabla_i\varphi\nabla_k\varphi, \tag{9}$$

where $\delta_{ik}^\perp = \delta_{ik} - l_i l_k$.

The energy minimum (8) is achieved by the solution $W = q_s z$, describing a system of equidistant smectic layers, orthogonal to the z axis. To describe deviations of smectic layers from this position, one should put $W = q_s(z - u)$, where u is the magnitude of the layer displacement vector along the z axis. Expanding the energy (8) in u, to second order we get the standard expression

$$E_W^{(2)} = \tfrac{1}{2}B(\nabla_z u)^2 + \tfrac{1}{2}K(\nabla^2 u)^2. \tag{10}$$

3. Dynamics of smectics C

We can now turn to the derivation of the equations for the long wavelength dynamics of smectics C. We shall largely follow [6, 10], but we shall retain a number of terms that were neglected there. The general form of the hydrodynamic equations is

$$\frac{\partial\phi_a}{\partial t} = \{\mathscr{H}, \phi_a\} - \hat{\Gamma}_{ab}h_b. \tag{11}$$

Here ϕ_a are a complete set of long wavelength variables, and a summation performed over the subscript b. $\{\mathscr{H}, \phi_a\}$ is the Poisson bracket with the hamiltonian (5). The differential operator $\hat{\Gamma}_{ab}$ in (11) is determined by a set of kinetic coefficients (of viscosity, thermal conductivity and permeability [5, 6]). h_a is a molecular field conjugate to ϕ_a:

$$h_a = \frac{\partial\mathscr{H}}{\partial\phi_a}. \tag{12}$$

For the orientational variable φ, for instance, the explicit form of the equation of motion following from (11) is

$$\frac{\partial\varphi}{\partial t} = -v_i\nabla_i\varphi - R_{ik}\nabla_i v_k - \Gamma h_\varphi, \tag{13}$$

where v is the velocity, Γ is the kinetic coefficient (Γ^{-1} has dimensions of viscosity and is analogous to the torsional viscosity of nematics), and

$$R_{ik} = \varepsilon_{lki}l_l + \lambda_1(n_{1i}l_k + n_{1k}l_i) + \lambda_2[n_{1i}\cdot(\mathbf{l}\times\mathbf{n}_1)_k + \mathbf{n}_{1k}\cdot(\mathbf{l}\times\mathbf{n}_1)_i]. \tag{14}$$

Here ε_{lki} is the totally antisymmetric tensor, and λ_1 and λ_2 are phenomenological parameters, describing the dynamic coupling of the orientational and undulation modes, mentioned in §1.

We also give here an explicit form for the viscosity tensor η_{iklm} (in the notation of [10]):

$$\eta_{iklm} = \eta_1 l_i l_k l_l l_m + \eta_2(\delta_{il}^{\perp}\delta_{km}^{\perp} + \delta_{kl}^{\perp}\delta_{im}^{\perp}) + \eta_3(l_i l_l \delta_{km}^{\perp} + l_k l_l \delta_{im}^{\perp} + l_i l_m \delta_{kl}^{\perp} + l_k l_m \delta_{il}^{\perp}) + (\eta_4 - \eta_2)\delta_{ik}^{\perp}\delta_{lm}^{\perp} + 2\eta_5(\delta_{lm}^{\perp} l_i l_k + l_l l_m \delta_{ik}^{\perp}). \quad (15)$$

Generally speaking, by virtue of the biaxiality of smectics C, the structure of the viscosity tensor in this phase is somewhat more complicated. However, owing to the small value of the tilt angle of the director to the normal **l**, we shall confine ourselves to the uniaxial expression (15).

4. Critical behaviour of S_C near $T_{S_C S_A}$

In this section we shall give a brief description of the behaviour of the parameters of the smectic C phase near $T_{S_C S_A}$, i.e. the temperature of the transition into the S_A phase. A detailed analysis of this problem can be found in [7, 11], which are devoted to a theoretical study of the S_C–S_A transition. Note that the conclusions arrived at in this investigation with regard to the S_C phase are to a considerable extent general, the point being that, owing to the small value of the tilt angle of the director in real smectics C, they can always be treated as being close to smectics A.

The order parameter describing the smectic C–smectic A transition is the vector ψ introduced in equation (2). The mean value of $\langle\psi\rangle$ is zero in the S_A phase and non-zero in the S_C phase. By virtue of its definition, $\langle\psi\rangle$ fixes an average tilt of the director **n** with respect to the normal to the smectic layer. It is straightforward to see that in the expansion of the free energy there are only *even* terms in ψ. Therefore the smectic C–smectic A transition is second order.

Because of the condition $\mathbf{l} \cdot \psi = 0$, the order parameter ψ has two independent components. However, the critical behaviour of a smectic at the S_A–S_C transition is not described by the standard $|\psi|^4$ model with two components. The order parameter ψ lies in a real but anisotropic space and is therefore coupled with vector quantities. This leads to a considerable critical dependence of the anisotropy of a smectic layer in the S_C phase. This manifests itself in pecularities of the critical dynamics of the transition [7].

Non-universality of the behaviour of the characteristics of a smectic at the S_C–S_A transition is evident in both the mean field theory and over a fairly broad region of strong critical fluctuations, where one can neglect corrections to the gradient term in the expansion of the free energy. In this region the critical behaviour of the parameters is described by non-universal indices, which are functions of α_1/α_2. The universal behaviour, associated with the $|\psi|^4$ model with two component order parameter, can occur only in a very narrow region near $T_{S_A S_C}$, where corrections to the gradient terms become important. The region of universal behaviour cannot apparently be reached experimentally; therefore we shall not dwell upon it. Thus henceforth the strong-fluctuation region implies the region of non-universal critical behaviour.

In the mean field theory in the S_C phase $\langle\psi\rangle^2 \propto T_{S_C S_A} - T$. In the region of strong fluctuations

$$|\langle\psi\rangle| \propto |T_{S_C S_A} - T|^{\beta}. \quad (16)$$

The exponent of the order parameter is $\beta = 0{\cdot}31$–$0{\cdot}5$. Above in (8) the smectic wave vector q_s has been introduced. The critical correction to it is determined by the law

$$\Delta q_s \propto \langle|\psi|^2\rangle. \quad (17)$$

Accordingly, in the mean field theory $\Delta q_s \propto T_{S_C S_A} - T$, and in the region of strong fluctuations $\Delta q_s \propto |T_{S_C S_A} - T|^{2\beta}$.

In the mean field theory quantities such as the heat capacity and compressibility of smectics experience an abrupt increase at the point of transition to the S_C phase. The smectic modulus B is particularly sensitive to the transition, decreasing by an amount of the same order of magnitude. In the strong fluctuation region there are critical contributions

$$\Delta C,\ \Delta B^{-1} \propto |T_{S_C S_A} - T|^{-\alpha} \tag{18}$$

to the heat capacity C and compressibility B^{-1}. These singularities are observed in both the S_A and S_C phases, but with different coefficients of $|T_{S_C S_A} - T|^{-\alpha}$. The heat-capacity index in (18) is small ($\alpha \lesssim 0{\cdot}1$); therefore it is difficult to observe this singularity experimentally.

Corrections to the modulus K that appears in the smectic energy are absent in mean field theory and are negligibly small in the strong fluctuation region. The moduli α_1, α_2 and α_3 appearing in the orientational energy (9) are given by

$$\alpha_1 = \tfrac{1}{4}K_1\langle\psi\rangle^2,\quad \alpha_2 = \tfrac{1}{4}K_2\langle\psi\rangle^2,\quad \alpha_3 = \tfrac{1}{4}K_3\langle\psi\rangle^2, \tag{19}$$

where K_1, K_2 and K_3 are the Frank constants, determining the energy of inhomogeneity of the director. The values of the moduli K_1, K_2 and K_3 can be approximated by the value of the modulus K; we therefore arrive to an estimate

$$\alpha_1,\ \alpha_2,\ \alpha_3 \approx K\langle\psi\rangle^2,$$

which determines the critical behaviour of the moduli α_1, α_2 and α_3.

The critical behaviour of the viscosity coefficients is quite different. The critical singularities in the coefficients determining the law of relaxation of the transverse velocity are absent, the viscosity coefficients η_1, η_4 and η_5 that determine the attenuation of first and second sound have strong critical behaviours. In mean field theory in the S_C phase (but not in the S_A phase) there are the following corrections to these coefficients:

$$\Delta\eta_1,\ \Delta\eta_4,\ \Delta\eta_5 \propto |T_{S_C S_A} - T|^{-1}.$$

In the strong-fluctuation region

$$\Delta\eta_1,\ \Delta\eta_4,\ \Delta\eta_5 \propto |T_{S_C S_A} - T|^{-\alpha - z\nu} \tag{20}$$

Here z is a critical dynamic index, which is close to 2, and ν is the correlation length index. In mean field theory $\nu = 0{\cdot}5$, in the strong fluctuation region $\nu = 0{\cdot}62$–$0{\cdot}64$. The fluctuational divergence (20) is observed in both the S_A and S_C phases [11] (but with different coefficients of $|T_{S_C S_A} - T|^{-\alpha - z\nu}$.

The critical behaviour of the kinetic coefficient appearing in (13) for the angle φ is determined as

$$\Gamma = 4(\gamma_1\langle\psi\rangle^2)^{-1}, \tag{21}$$

where γ_1 is the torsional viscosity coefficient, for which there is an estimate $\gamma_1 \approx \eta_2$, η_3. Note that (21) is in good agreement with the well-established but still rather surprising experimental observation that the time of orientational relaxation in smectics C and C* is determined by an effective viscosity that is an order of magnitude smaller than that in smectics A. For the reactive phenomenological parameters λ_1 and λ_2 in (14) one can obtain expressions [7]

$$\lambda_1 = \frac{\lambda + 1}{|\langle\psi\rangle|},\quad \lambda_2 = \lambda. \tag{22}$$

Here λ is a nematic reactive parameter close to unity for substances that consist of rod-like molecules. We should stress that the quantities γ_1 and λ have no critical singularities.

The estimates given here are valid for the low frequency (hydrodynamic) region. If the characteristic frequency ω exceeds the inverse time of order parameter relaxation then the critical corrections to the parameters studied by us acquire considerable frequency dependences. We shall not give the respective estimates here but shall note only that the frequency dependence suppresses both the mean field and fluctuational contributions.

5. The low frequency mode

The presence of the orientational degree of freedom in smectics C gives rise to an extra low frequency mode (compared with the S_A-phase). This mode has a diffusive character and is analogous to the director mode. As in nematics, this mode is slow, which is accounted for by a small value of the parameter

$$K\varrho/\eta^2 \ll 1,$$

where ϱ is the density, K is a quantity of the order of magnitude of the Frank modulus, and η is the characteristic viscosity.

In smectics second sound also propagates. For propagation angles close to 0° or 90° with respect to the smectic layers, this acoustic mode splits into two diffusional modes. One is fast and is associated with shear velocity relaxation. The other is slow and is associated with relaxation of smectic layer displacements. It is with this (undulation) mode that fluctuational effects in smectics are associated.

The explicit form of the system of equations from which one can obtain the dispersion law for the orientational and undulation modes can be found from (11) by means of simple but rather cumbersome calculations. Details can be found in [5, 6], and we shall give only the results here, taking into consideration the fact that, as follows from (22), $\lambda_1 \gg \lambda_2 \approx 1$. In the region of wavevectors $q_z \ll \eta_3 q_\perp^2/(B\varrho)^{1/2}$ that is important for us, the system of equations obtained describes two branches of the spectrum, corresponding to the smectic and orientational modes. The dispersion laws for these branches are

$$\omega_\pm(\mathbf{q}) = \tfrac{1}{2}(\omega_1 + \omega_2) \pm \tfrac{1}{2}\left[(\omega_1 - \omega_2)^2 + \frac{q_y^2}{q_\perp^2}\frac{4m\omega_1\omega_2}{1 + mq_y^2/q_\perp^2}\right]^{1/2}. \tag{23}$$

Here the notation

$$m = 2\lambda_1^2/\Gamma\eta_3, \tag{24}$$

$$\omega_1 = -i\frac{Bq_z^2 + Kq_\perp^4}{\eta_3 q_\perp^2}, \quad \omega_2 = -i\Gamma\left(1 + m\frac{q_y^2}{q_\perp^2}\right)\alpha q^2 \tag{25}$$

is introduced. By virtue of the inequality $q_z \ll q_\perp$, one should put $\alpha q^2 = \alpha_1 q_x^2 + \alpha_2 q_y^2$ in (25) (the y axis is oriented along the vector $\mathbf{n}_1$).

Inserting the explicit expressions for Γ and λ_1 into the definition (24), we get

$$m = 4(1 + \lambda)^2\gamma_1/\eta_3.$$

Bearing in mind that $\eta_3 \approx \gamma_1$ and $\lambda \approx 1$, we conclude that the quantity m is a dimensionless parameter of order unity. For $Bq_z^2 \approx Kq_\perp^4$ the frequencies ω_1 and ω_2 are

of the same order of magnitude and do not depend on the closeness to the point of transition to the S_A phase.

As follows from (23), the parameter m determines the coupling of the orientational and undulational modes. Because $m \approx 1$ and $\omega_1 \approx \omega_2$, this coupling is strong. However, for $Bq_z^2 \gg Kq_\perp^4$ this coupling effectively vanishes and the dispersion law $\omega_-(\mathbf{q})$ acquires the form

$$\omega = -i\Gamma\alpha q^2, \tag{26}$$

where now $\alpha q^2 = \alpha_1 q_x^2 + \alpha_2 q_y^2 + \alpha_3 q_z^2$. Thus the expression for $\omega_-(\mathbf{q})$ can be employed for defining the spectrum of the orientation mode for any wavevectors.

Note that the spectrum of the orientation mode in a hexatic smectic B, where the coupling of the orientational and undulation modes is absent, does not coincide with (26). The point is that in the derivation of the dispersion law (26) we have used the inequality $\Gamma \gg \eta_2^{-1}, \eta_3^{-1}$, which in hexatic smectics B is not satisfied. The spectrum of the orientational mode in smectics B can be found in [6].

6. Smectic layer fluctuations

Dynamic effects associated with fluctuations of smectic layers can be conveniently studied by means of the diagram technique. We shall use the formalism proposed in [12].

Owing to fluctuations in smectic layers, there appear corrections to the dispersion law for the spectrum of slow modes (23) derived in the linear approximation. This effect is accounted for by self-interaction of the smectic mode, which is described by those terms in the dynamical equations that are non-linear in the displacement vector. Fluctuational corrections to the equations of dynamics for weakly fluctuating variables appear when the interaction of the modes is incorporated.

As has been mentioned above, the single strongly fluctuating degree of freedom of a smectic is associated with displacements of smectic layers. This means that one should take into account only non-linearities in the displacement vector. As the result of a rather complicated analysis (analogous to that performed in [5, 6] for smectics A and B), we obtain the following expressions for the fluctuational contributions to the viscosity coefficients:

$$\Delta\eta_1 = 2(1+\gamma)^2 A, \quad \Delta\eta_4 = 2\gamma^2 A, \quad \Delta\eta_5 = 2\gamma(1+\gamma)A, \tag{27}$$

where

$$A = \frac{T}{|\omega|}\frac{B^{3/2}}{K^{3/2}}\frac{1}{128}\left\{1 + \frac{8m}{\pi^2}\int_0^{2\pi} d\theta \int_0^{\pi/2} d\varphi \cos^2\theta\,(1 + m\cos^2\theta)\cos^4\varphi \right.$$
$$\left. \times \left[1 + \frac{9(I_+^4 + I_-^4) + 18(I_+ I_-)I_-^2 + 44(I_+ I_-)^2}{(I_+ - I_-)(9I_+^2 - I_-^2)(3I_- + I_+)}\right]\right\}. \tag{28}$$

Here the following notation has been introduced:

$$I_\pm = \left[1 \pm \left(1 - \frac{4f\cos^2\varphi}{(1 + m\cos^2\theta)(1 + f\cos^2\varphi)^2}\right)\right],$$
$$f(\cos\theta) = (1 + m\cos^2\theta)[m_1 + (m_2 - m_1)\cos^2\theta],$$
$$m_{1,2} = \alpha_{1,2}\lambda_1/Km.$$

The dimensionless coefficient γ is determined by

$$\gamma = -\left(\frac{\partial \ln q_s}{\partial \ln \varrho}\right)_\sigma. \tag{29}$$

The main conclusion concerning the dynamics of smectics follows from these expressions: the divergence of the bulk viscosity coefficients due to fluctuations is proportional to $|\omega|^{-1}$.

The expression (28) holds for the smectic C phase, where the parameters m, m_1 and m_2 determining the function f are of order unity. The respective formula for the S_A phase is obtained from (28) by putting $m = 0$. Note that it follows from the explicit form of (28) that the coefficient of $B^{3/2}/K^{3/2}|\omega|$ for the S_C phase always exceeds that for the S_A phase.

For information, we shall give the spectrum of first and second sound with the fluctuational contribution taken into account:

$$\left.\begin{aligned} \omega &= \pm C_1 q - \frac{i}{\varrho q^2}\left[\tfrac{1}{2}(\eta_2 + \eta_4)q_\perp^4 + (\eta_5 + 2\eta_3)q_z^2 q_\perp^2\right. \\ &\quad \left. + \tfrac{1}{2}\eta_1 q_z^4 + A(\gamma q^2 + q_z^2)^2\right]; \\ \omega &= \pm C_2 q - \frac{i}{\varrho q^2}\left[\tfrac{1}{2}\eta_3(q_z^2 - q_\perp^2)^2\right. \\ &\quad \left. + \tfrac{1}{2}(\eta_1 + \eta_2 + \eta_4 - 2\eta_5)q_z^2 q_\perp^2 + A q_z^2 q_\perp^2\right]. \end{aligned}\right\} \tag{30}$$

Here the bare viscosity coefficients (ω independent) are involved, and the velocities of first, C_1, and second, C_2, sound are defined by

$$C_1^2 = \left(\frac{\partial P}{\partial \varrho}\right)_\sigma, \quad C_2^2 = \frac{B}{\varrho}\frac{q_z^2 q_\perp^2}{q^4}. \tag{31}$$

By virtue of $A \propto \omega^{-1}$ in the low-frequency limit, the fluctuational damping of the sounds exceeds the bare damping, which means that the terms with η in (30) can be neglected compared with A.

7. Conclusions

We have been interested in differences in the attenuation of the sound in smectics A and smectics C. In the S_C phase, there is an extra orientational mode compared with the S_A phase. This by itself does not affect the expression for the fluctuational attenuation. Therefore in hexatic smectics B, where the orientational mode is also present, the expression for the fluctuational attenuation of sound has the same form as for smectics A [6]. However, in smectics C the situation becomes more complicated.

The fluctuational attenuation appears in the region of the existence of the orientational mode, which is strongly coupled with the undulation mode in smectics C owing to their lower symmetry compared with hexatic smectics B. In particular, this leads to distortion of the spectrum of this mode for wavevectors with small q_z. We shall stress that the strong coupling of the orientational and undulation modes takes place despite the weak anisotropy of the layers in real smectics C.

The presence of the strong coupling of the modes does not affect the main conclusion that the bulk viscosity coefficients diverge as ω^{-1} owing to fluctuations. However, it does lead to different coefficients of ω^{-1} in the S_C and S_A phases. Explicit

expressions for fluctuational corrections to the viscosity coefficients in smectics C are given by (27) and (28). The integral appearing in (28) can only be found numerically.

Note that the fluctuational corrections to the viscosity coefficients η_1, η_4 and η_5 only are large. For the viscosity coefficient η_3 there are only weak logarithmic corrections, and there are no corrections at all to the viscosity coefficient η_2 or to the thermal conductivity and permeability coefficients.

In conclusion, let us compare our theoretical picture with experimental data obtained by Collin *et al.* [8, 9] for the smectic A and C phases of the same substance. In particular, we note that the modulus B in the S_C phase proves to be smaller than that in the S_A phase, which agrees with the results of the analysis of the critical behaviour of B at the smectic C–smectic A transition (see §4). However, Collin *et al.* paid most attention to the analysis of the fluctuational attenuation of sound. The angular dependence of this attenuation led Collin *et al.* to the conclusion that the dimensionless coefficient γ in (27) and (30) has not only a different value but also a different sign in the S_A and S_C phases (as a result of which, the fluctuational attenuation of first sound in the S_C phase becomes zero for a certain propagation direction). There is nothing surprising in this. From the definition (29) and the law (17) it follows that in mean field theory the quantity γ changes abruptly at the S_C–S_A transition by an amount of the same order of magnitude. In the strong-fluctuation region

$$\gamma \propto |T_{S_C S_A} - T|^{2\beta-1}.$$

We should stress that the coefficients of $|T_{S_C S_A} - T|^{2\beta-1}$ have different signs in the S_A and S_C phases, which is in agreement with the experimentally observed behaviour of γ.

Analysing the fluctuational attenuation of sound in the S_C phase, Collian *et al.* [8] concluded that the value of the modulus K in the S_C phase is smaller than that in the S_A phase. However, our investigation (see §4) has shown that the corrections to K in the first approximation are absent, and consequently the values of K in the S_A and S_C phases should be not noticeably different. This inconsistency is easily removed on noting that in [8] Collin *et al.* analysed the experimental data for the S_C phase using an expression for the fluctuational attenuation that is valid only for the S_A phase. Our expression (28) for the fluctuational contribution to the viscosity coefficients of the S_C phase is always larger than that for the S_A phase. This accounts for the apparent decrease is the modulus K noted by Collin *et al.* [8].

Thus the theory that we have constructed shows that in the long wavelength dynamics of all types of smectics, fluctuational effects are associated with the undulation mode. In addition, in smectics C, owing to their lower (compared with smectics A and B) symmetry, an important role is played by the dynamic anisotropy of smectic layers, leading to coupling of the orientational and undulation modes. When interpreting experimental data (such as those concerning the sound absorption) together with these conclusions, one should also take into consideration the relative narrowness of the temperature range for the existence of smectic phases, i.e. their closeness to the smectic A–smectic C or smectic B–smectic C phase transition points.

References

[1] LANDAU, L. D., and LIFSHITZ, E. M., 1980, *Statistical Physics* (3rd edition revised E. M. Lifshitz and L. P. Pitaevskii (Pergamon).

[2] GRENSTEIN, G., and PELCOVITS, R. A., 1981, *Phys. Rev. Lett.*, **47**, 856.

[3] KATS, E. I., 1982, *Soviet Phys. JETP*, **56**, 791.
[4] MAZENKO, G. F., RAMASWAMY, S., and TONER, J., 1983, *Phys. Rev.* A, **28**, 1618; see also the correction in MILNER, S. T., and MARTIN, P. C., 1986, *Phys. Rev. Lett.*, **56**, 77.
[5] KATS, E. I., and LEBEDEV, V. V., 1983, *Soviet Phys. JETP*, **58**, 1172.
[6] KATS, E. I., and LEBEDEV, V. V., 1985, *Soviet Phys. JETP*, **61**, 484.
[7] GUROVICH, E. V., KATS, E. I., and LEBEDEV, V. V., 1988, *Zh. éksp. teor. Fiz.*, **94**, 167.
[8] COLLIN, D., GALLANI, J. L., and MARTINOTY, P., 1986, *Phys. Rev. Lett.*, **58**, 954.
[9] COLLIN, D., GALLANI, J. L., and MARTINOTY, P., 1986, *Phys. Rev.* A, **34**, 2255.
[10] MARTIN, P. C., PARODI, O., and PERSHAN, P. S., 1972, *Phys. Rev.* A, **6**, 2401.
[11] TABIDZE, A. A., ABRAMKIN, G. P., KOSHKIN, N. I., 1988, *Pis'ma Zh. éksp. teor. Fiz.* **47**, 386. FAFHO, F., MATSUSHITA, M., and SAWATA, Y., 1982, *Phys. Lett.*, **93A**, 27.
[12] KHALATNIKOV, I. M., LEBEDEV, V. V., and SUKHORUKOV, A. I., 1984, *Physica* A, **126**, 135.